DETERMINATION OF DAMPING PROPERTIES OF LAYERED STRUCTURES BY MEANS OF WAVE-PROPAGATION-BASED METHODS

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1. Introduction

Viscoelastic materials exhibit high internal damping. This feature is exploited in their industrial applications ranging from aircraft constructions to submarines and land vehicles. High interest in those materials is reflected in number of publications devoted to damping by means of viscoelastic tapes and films. Much attention is paid to tranverse vibrations of laminated beams with viscoelastic layer (cf. [1, 2]). Since response of such multilayer systems depends to a high degree on a damping parameter it seems desirable to develop solutions enabling its determination.

Existing methods of damping identification can be divided into two groups. Modal formulation is grounded on calculation of half-power bandwidth [1]. Due to drastic increase of modal density in higher frequencies, observed in all structures, the approach is limited to low band covering first few modes of vibrations. Moreover, only values of damping parameter, corresponding to natural frequencies, can be estimated by means of the method. The other class of solutions is based on analysis of wavefield generated in vibrating structure. The k-space methods take into account damping by introduction of complex wave number.

The paper presents results of experimental verification of Inhomogenous Wave Correlation method [3, 4], obtained for cantilever beam covered with viscoelastic layer. Dependence of complex wave number on frequency is investigated. Results obtained by means of alternative, older solution, proposed by McDaniel et al [5], are also demonstrated. Section 2 and 3 show theoretical bases of IWC and MacDaniel method, respectively. When compared to original idea, two significant modifications are introduced to the last method. First, random continuous excitation instead of transient one has been used. Second, similarly to what had been done in IWC method, coherence function has been employed in identification algorithm. Both methods use results of the same experimental procedure described in section 4. Wavefield generated in a sample is reconstructed from measurements done in points located along its length. Comparison of experimentally obtained transfer function with theoretical solution enables determination of complex wave number in terms of frequency. Identification algorithms implemented in MATLAB are used to find the parameter. The paper concludes with discussion of results and final remarks.

2. Theoretical formulation of IWC method

IWC method was originally developed as a tool for identification of direction-dependent dispersion curves in plane structures. As a one-directional propagation is assumed, description of a wavefield simplifies significantly and takes form:

$$W(x) = e^{i\bar{k}x} \tag{1}$$

for given frequency ω, where $\bar{k}$ is complex. Relation between wave number and loss factor η is given by ratio of phase and group velocity, denoted by c_φ and c_g, respectively [6] :

$$\frac{\mathrm{Im}\{\bar{k}\}}{\mathrm{Re}\{\bar{k}\}} = \eta \frac{c_\varphi}{2c_g} . \tag{2}$$

Since for bending waves group velocity is twice the phase velocity, loss factor may be expressed directly in terms of wave number:

$$\eta = 4 \frac{\mathrm{Im}\{\bar{k}\}}{\mathrm{Re}\{\bar{k}\}} . \tag{3}$$

Correlation between wave observed in experiment and the one described theoretically is calculated for every frequency by means of the formula:

$$IWC(\bar{k}) = \frac{\left| \sum_{i=1}^{n} W^*(x_i, \bar{k}) \cdot \tilde{W}(x_i) \rho(x_i) \right|}{\sqrt{\sum_{i=1}^{n} \left| W(x_i, \bar{k}) \right|^2 \sum_{i=1}^{n} \left| \tilde{W}(x_i) \right|^2 \rho(x_i)}} , \tag{4}$$

where $\tilde{W}(x_i)$, $\rho(x_i)$ stand for transfer function and coherence, respectively, both calculated for response at x_i, * denotes complex conjugate and x_i is a location of ith measurement point. Value of $\bar{k}$ is determined by maximisation of ratio (4). Finite set of pairs $(\mathrm{Re}\{\bar{k}\}, \mathrm{Im}\{\bar{k}\})$ is searched for solution, no optimisation algorithm is used.

3. Theoretical formulation of McDaniel method

The method is based on analysis of vibrations of Bernoulli beam. Homogenous governing equation is used:

$$EJ\frac{\partial^4 w(x,t)}{\partial x^4}+\rho A\frac{\partial w(x,t)}{\partial t^2}=0\,. \tag{5}$$

After being Fourier transformed, the equation (5) becomes:

$$E\big(1-i\,\eta(\omega)\big)J\,\frac{\partial^4 W(x,\omega)}{\partial x^4}-\omega^2\rho A W(x,\omega)=0\,, \tag{6}$$

where introduced term $E\big(1-i\,\eta(\omega)\big)$ describes damping. Solution

$$W(x,\omega)=c_1(\omega)e^{i\bar{k}x}+c_2(\omega)e^{-i\bar{k}x}+c_3(\omega)e^{\bar{k}x}+c_4(\omega)e^{-\bar{k}x} \tag{7}$$

includes complex wave number:

$$\bar{k}=\sqrt[4]{\frac{\rho A}{E\big(1-i\,\eta(\omega)\big)J}\,\omega^2}\,. \tag{8}$$

If the theoretical model is correct, relation (7) should be satisfied for every measured response. Thus, a set of equations written in matrix form for given frequency ω

$$\begin{bmatrix} W(x_1) \\ .. \\ W(x_i) \\ .. \\ W(x_n) \end{bmatrix}=\begin{bmatrix} e^{i\bar{k}x_1} & e^{-i\bar{k}x_1} & e^{kx_1} & e^{-kx_1} \\ .. & .. & .. & .. \\ e^{i\bar{k}x_i} & e^{-i\bar{k}x_i} & e^{kx_i} & e^{-kx_i} \\ .. & .. & .. & .. \\ e^{i\bar{k}x_n} & e^{-i\bar{k}x_n} & e^{kx_n} & e^{-kx_n} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \tag{9}$$

ought hold, where x_i , $i=1,..,n$ denotes a location of ith measurement point and $W(x_i,\omega)$ is a response obtained at x_i. Unknown constants c_j , $j=1,..,4$ are determined by means of least square method for each complex value of $\bar{k}$ assumed in consecutive loops of optimisation procedure. Original objective function, proposed by McDaniel et al, has been modified. Now it introduces coherence function ρ :

$$\varepsilon(\bar{k})=\frac{\sqrt{\sum_{i=1}^{n}\left|W(x_i,\bar{k})-\widetilde{W}(x_i)\right|^2\rho(x_i)}}{\sqrt{\sum_{i=1}^{n}\left|\widetilde{W}(x_i)\right|^2\rho(x_i)}}\,, \tag{10}$$

where $\widetilde{W}(x_i)$ denotes experimentally obtained response. MATLAB built-in routine based on Nelder-Mead simplex algorithm [7] has been employed. Loss factor may be calculated from expression (8):

$$\eta = \frac{\mathrm{Im}\{\bar{k}^4\}}{\mathrm{Re}\{\bar{k}^4\}} \quad , \tag{11}$$

if $\bar{k}$, minimizing (10), is used.

4. Experimental validation

Experimental apparatus is depicted at figure 1. Steel sample of dimensions 0.27m×0.02m×0.001m covered with viscoelastic material manufactured by RIETER France was tested . Clamping was realized by means of massive vice. Gearing&Watson V4 shaker with Bruel&Kjaer 8200 force charge mounted on its head, excited the sample at the free end. Polytec OFV 350 laser vibrometer measured velocity in 26 points spaced 0.01m apart along length of the beam. Bruel&Kjaer Pulse multi-analyzer system acquired data.

Fig. 1. Experimental apparatus

Frequency dependence of real and imaginary part of wave number obtained for both presented methods is shown in figures 2 and 3, respectively. Shape of the curves representing real part of wave number agrees very well with the one predicted theoretically within the whole frequency range. Both methods produce almost identical results. However, evident differences are observed in imaginary part of wave number. Loss factor calculated with the aid of formulas (3) and (11) for IWC and McDaniel method, respectively, is plotted in figure 4. The fact that the parameter determined by means of

McDaniel procedure takes negative value is a consequence of using minus sign in a damping term appearing in equation (6). If loss factor of opposite sign was obtained and the notation stayed unaltered, erroneous model of dissipation of energy would result (cf. [8]). Finally, it is observed that an average value of the parameter does not change with frequency. This means that hysteretic model of damping applies. Unfortunately, no simple physical interpretation exists for loss factor obtained by use of IWC method.

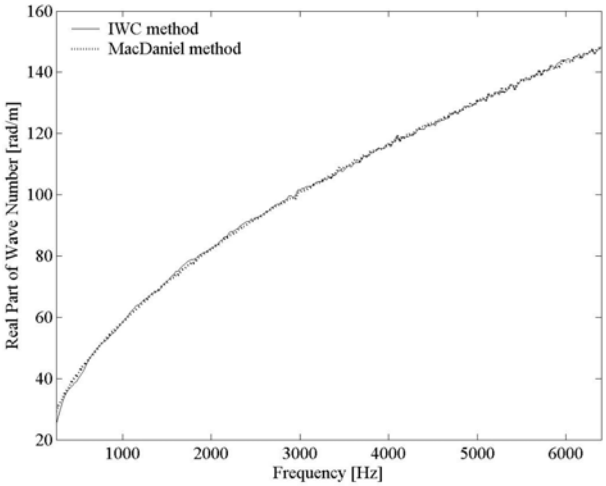

Fig. 2. Real part of wave number vs frequency

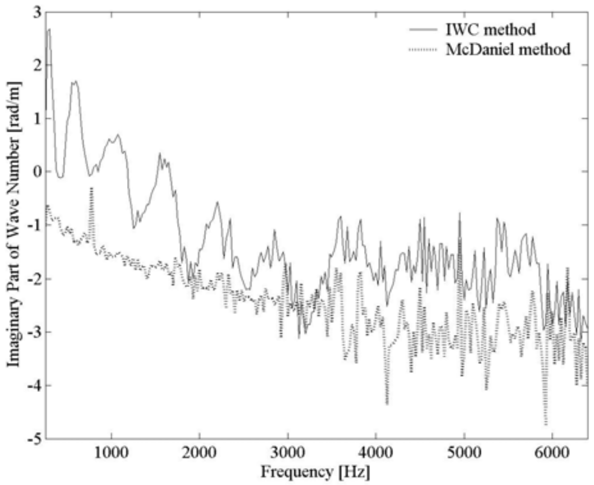

Fig. 3. Imaginary part of wave number vs frequency

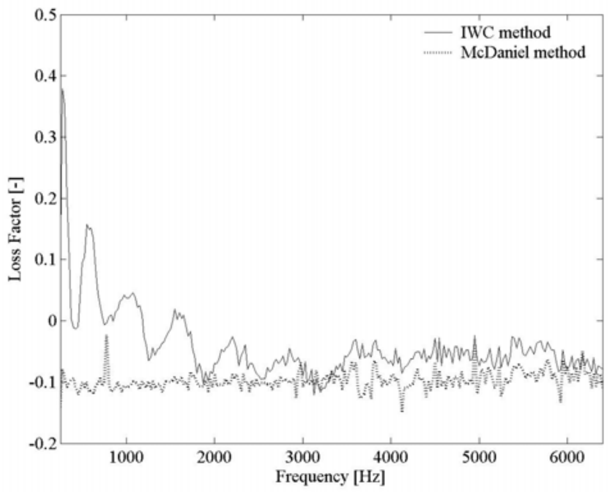

Fig. 4. Loss factor vs frequency

5. Conclusions

Two methods of damping identification, both based on wave approach, have been verified experimentally. Beam sample covered with viscoelastic layer has been used in tests. Method proposed by McDaniel et al was originally validated for low frequencies ranging up to 500Hz. The limitation resulting from appearance of other types of waves has been removed in the described experiment. Loss factor has been determined within 6.4kHz bandwidth. IWC method adapted to the case of one-dimensional wavefield has been used as an alternative to approach developed by McDaniel et al.

Foregoing analysis lacks some important issues which will be addressed in extended version of the paper. Missing considerations embrace, among others, profound discussion on obtained parameters, comparison of the results with modal data and study on limitations of the methods.

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